The Special Formula of the Maximum Impact Force

P.P. de Lima-e-Silva
www.ceu-aberto.com
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ABSTRACT

This paper introduces improvements in the classic formula of the maximum impact force (F_{MAX}) on a falling rock climber. The formula is said special because it is restricted to free fall, without contact with the rock. The Experimental Model 1, or EM1, is a semi-empirical model, it takes into account two relevant factors ignored by the classic Wexler model, the dynamic belay (DB) and the friction on to the last carabiner (LCF). The EM1 adds to F_{MAX} a formula for the maximum force onto the last anchor (F_{BOLT}), a concern of the present time. The study has sought to explain the apparent paradox between estimated F_{BOLT} values supposedly higher than the P-bolts resistance used in Brazil. The methodology has included a historical summary of the classic formula, study of literature, observations from experience, and a short sensitivity analyses. The comparison between F_{MAX} and F_{BOLT} values estimated with the classic and EM1 models has revealed strong differences, and may explain the absence of severe accidents in Brazil due to P-bolt failure. The conclusion is that EM1 equations produce F_{MAX} and F_{BOLT} values remarkably lower and, according to the accident records, more realistic, allowing better estimates of the risks.

keywords: climbing forces, fall factor, maximum impact force, rock climbing, climbing safety review

1.0 CONTEXT

1.1 Introduction

Climbing is a risky activity. Free rock climbing poses inherent risks, and it is impossible to control all the variables. The mental and physical condition of the climber, the elements of distraction, the ever-changing environmental and rock conditions, degree of difficulty, quality of used material, all of them elements difficult, but not impossible, to control. The purpose of this study is not to eliminate the inherent risks that are the soul of the sport, but to enable Brazilian climbers to be aware of the risks by means of greater knowledge of the climbing physics and the forces that work in this eminently technical sport, and so make them able to control those risks.

The maximum impact force on the falling climber (F_{MAX}) is the basic parameter to establish the safety of the protection system for free climbing practitioners, the climbing style in which all safety equipment used, such as rope, harness, tapes, carabiners, and braking devices have the sole function of operating if an environmental or human failure occurs causing a climber to fall. It is important to note that in this paper we will always refer to dynamic ropes, since they are the ropes used for free rock climbing and applied to belayed climbing.

The expression "maximum force" arises because during a fall the force on the rope and thou on the climber varies in time, starting with a minimum value, going to a maximum one and retreating to another minimum, the climber's weight. The chart of Figure 1 shows this variation during the fall. It is an adaptation of a laboratory monitored fall-chart, where the force on the climber was recorded with the precision of hundredths of a second, from the release of the weight until its final immobilization (Czermin *et al.*, 2007, p.11). During that test, the force reached its maximum value of 7kN in about 0.14s after the rope started elongating.

In Phase 1 of the fall the climber is falling, but the open rope has not yet been fully extended. The force on it is only its own weight. In Phase 2, now fully extended, it starts to stretch like a spring. The weight of the falling climber, accelerated by gravity, continues to act but the rope begins to resist, progressively, which causes a corresponding increase of the force on the rope, and on the entire system, to the point where it reaches its maximum stretching point, when the climber stops falling; at that very moment, the force on the rope is maximum.

The movement then enters Phase 3; due to the elastic characteristic of the rope, it begins to recede and the elastic energy accumulated therein, the fraction not consumed by the internal friction, is restored in a reverse movement to a new position of balance, where the force decreases until it comes to an equilibrium with the climber's weight, making the climber motionless. Note that if the rope were in fact a spring, the climber would go into an oscillatory motion, bouncing up and down until the friction consumed all the energy and it stopped. But this is not the case, because a striking feature of the climbing rope comes into play, its strong damping effect by the internal friction which quickly takes the climber to a "dynamic" state of rest[1], usually even without a second swing, except some small oscillations caused by the climber's movements seeking for a more stable position.

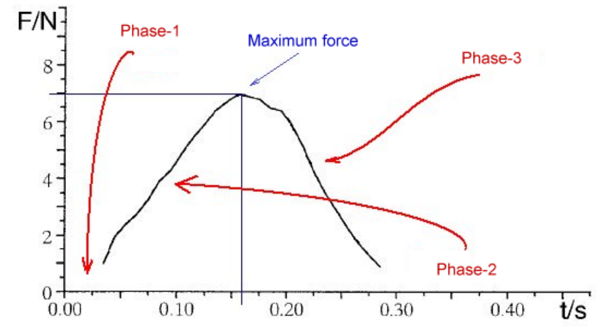

Figure 1 – Typical variation of the impact force in time for a rope leader fall. Chart adapted from a real case of laboratory experiment (Czermin *et al.*, 2007, p,11).

Thus, maximum force is not the greatest force that can be exerted in any fall, but the greatest force exerted on the climber between the moment he/she falls, until the moment he/she is brought to rest by the safety system. The chart of Figure 1, despite being the result of a specific experiment, is representative of the general behaviour of the force in a fall, going from a minimum, up to a maximum value and back to a different minimum. However, the force may reach higher or lower maxima in a free fall, depending mainly on the weight of the climber, the fall factor, the friction at play, the dynamic belay and the rope's elongation capacity.

1.2 Objectives

- **Main objective**

The ultimate objective of this study is to obtain a formula for maximum impact force onto the last anchor (F_{BOLT}), a modern concern, because it is this value that is going to determine the tension that the anchor should support in case of a fall. But as most of literature focuses on F_{MAX}, the study starts from there, taking into account that F_{BOLT} is built upon F_{MAX} mathematical model. The improvements targeted are firstly to include the dampness effect of the dynamic belay (DB), which strongly affects both F_{MAX} and F_{BOLT}, and secondly, considering the friction between rope and the last carabiner (LCF), that affects slightly F_{MAX} and strongly F_{BOLT}. These two mathematical models constitute what is called herein Experimental Model 1, or just EM1.

All free climbing ropes manufactured today, that carry the UIAA[2] label, necessarily comply with the maximum impact force limit of 12kN on the falling climber's body, i.e. $F_{MAX} \leq 12kN$[3]. In fact, the climbing ropes, with a few exceptions, carry an F_{MAX} certified mark below 9kN, and the vast majority in the 7-9kN range. As the ropes resist to much higher tensions (18-23kN), it is impossible, under normal conditions, for a rope in good shape to break due to excessive force, even in the most severe fall, a factor-2 fall. (See UIAA rope test configuration on Figure 2.)

This fact is well known to physicists and engineers climbers, such as Leuthäusser (2016a, p.5), whose assessments of rope strength and stretching abilities have led him to reaffirm that:

> *"…The highest possible maximum force (with m=80kg and the highest possible fall factor 2) is only 6% greater than the impact force on a UIAA standard fall. With a tensile strength of about 20kN, i.e. more than twice as necessary, a climbing rope can not be broken only by sheer extension in a normal fall (without a cutting edge to cut the rope).*

This paper restricted itself to the analysis of the phenomenon for single ropes, because double ropes cause forces smaller than the single ones, and the twin ones are rarely used in Brazil. It also does not address special cases

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[1] Actually, there is always some oscillation, because the climber moves and the rope is elastic, but these oscillations are small and does not affect F_{MAX}.
[2] Union Internacionale des Association d'Alpinisme, the international certifier for mountaineering equipment, with headquarters in Switzerland.
[3] The value is based on studies of the American Army around 1950's to define the maximum reasonable force that the parachute can impose to a human body without causing any detectable injury.

such as rope breakage accidents by fission in a cutting slit or other incidental reasons, focusing on F_{MAX} to obtain F_{BOLT}, as the latter can cause the anchor failure and therefore impose risk of death.

- **Secondary objectives**

In addition, this paper aims to achieve three secondary objectives: [a] the original version in Portuguese aimed to provide a bibliographical-historical review for Portuguese-speaking climbers on the physics of climbing papers published originally in English; [b] to confront the proposed models of calculation, in fact variations of the basic, classical formula with the real phenomenon, seeking explanatory clues of the apparent contradictions between the values calculated by the classical formula, the resistance of the anchors as reported on non scientific documents, and the absence of accidents due to P-bolt failure in Brazil; [c] to identify the factors absent in the classical formula, to analyze the possibilities of quantification of those factors, and propose theoretical or semi-empirical formulations that can consider them in a practical manner.

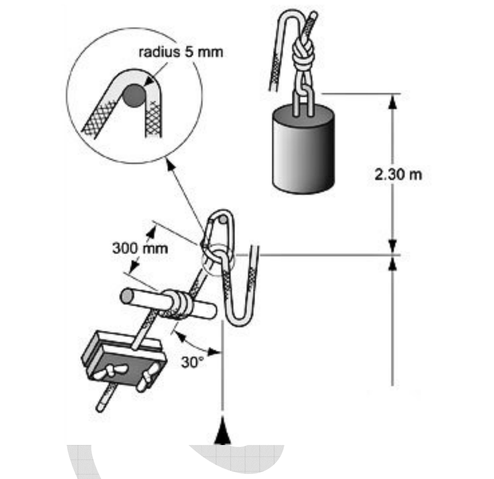


Figure 2 – Schematic details of the UIAA test for dynamic ropes.

The simplified formulation of F_{MAX}, because it ignores slack rope and other details, does not involve sophisticated mathematics and physics, and can be understood by anyone with basic training in mechanics (physics) and algebra. But there are complexities within simplicity, a characteristic of objective reality. Some factors of these complexities will be addressed in the final part of this paper, since they play a relevant role both in intensification and attenuation of the intervening forces, which must be known to the climbers in order to enable them to better manage their risks.

This paper was not made to be precise in the determination of the forces during a climber's fall, a reasonable approximation is acceptable. The underneath purpose of these descriptions and analysis is to bring about that kind of information to Brazilian climbers, making this paper the first of a series about climbing safety, aiming to motivate others Brazilian climbers to produce more studies upon the Physics of rock climbing.

2.0 BIBLIOGRAPHICAL REVIEW

A literature review shows that since the first known publication by Arnold Wexler in 1950, little progress towards a more realistic formula of F_{MAX} has been made (Wexler, 1950). As climbing physics is, in short, relatively simple, it was expected to be so. But the complexity of modelling the non-linear behaviour of the rope modulus[4] – which determines its elongation capacity – pose challenges. In addition, there are external factors of difficult generalization that exert strong influence on F_{BOLT}. These aspects make it difficult to model other diffuse variables, such as friction between the rope and the last carabiner, considering the hundreds of models of carabiners available, as well as the techniques of dynamic belay, when there are uncounted ways to do it, to cite the diversity of only two of those variables.

2.1 Wexler (1950): The Theory of Belaying

Arnold Wexler was the pioneer, earliest known record researcher who addressed the Physics of climbing, both by introducing the F_{MAX} equation and by creating the fall factor concept (without this denomination, then). His paper, published on the American Alpine Journal in 1950, is only preceded by Wexler's own 1946 essay published in a Sierra Club bulletin, no longer available (Jarvis, 2017). The interesting thing is that Wexler was not trying to assess F_{MAX}, he presented the formula because he wanted to show that the dynamic belay (DB) was important to reduce F_{MAX}, and thus escape the risk of breaking the rope.

In those times the ropes had much less elastic capacity and therefore less dynamic resistance and damping capacity than today, justifying the concern. To derive the formula, he assumed that the rope obeys Hooke's Law (elasticity), producing the formulation of Equation 2-1, and its derivation is in Table 1.

[4] Note that different researchers attribute to the word "module" different things. Ulrich Leuthäusser, as well as the website of the Beal rope, call "modulus" the Young's modulus (Leuthäusser, 2016a; Beal, 2004), the constant material of the rope that embodies intrinsic structural physical characteristics (N/m²); others (e.g. Attaway, 1996; Wexler, 1950) attribute to the "rope module", or simply "module", the letter "K" or "M", which represents the product of the Young's modulus by the section area of the rope (E•A), with force dimension. In order to avoid confusion, in this paper we assign to the Young's module (E) its own name, and assign "module" to the product E•A (K or M symbol).

$$P = W + W\sqrt{1 + \frac{2kH}{WL}}$$

[Eq.2-1]

Figure 3 – Maximum force formula as it appears in Wexler (1950, p.4). P is the maximum force, W is the weight of the climber, H is the height of the fall, L is the amount of open rope[5], and k is the modulus of the rope, which includes Young's modulus and its cross section. The author alerts that the formula disregards dynamic belay.

Wexler was also a pioneer in pointing out the maximum value of the fall factor[11]. A fall factor 2 occurs when the ratio of fall height to open rope length is maximum, i.e., when the energy per rope unit is maximized. This situation occurs when the climber falls before doing the first clip; if he/she climbs to a height of H meters and falls, his fall will be of 2H meters, and the amount of open rope will be H meters. By dividing the height of the drop by the open rope, 2H/H=2 is obtained, the theoretically largest possible fall factor under normal conditions (Figure 4).

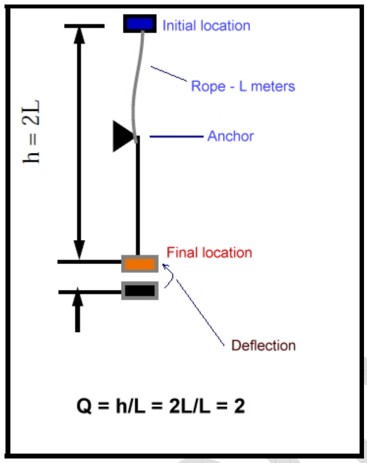
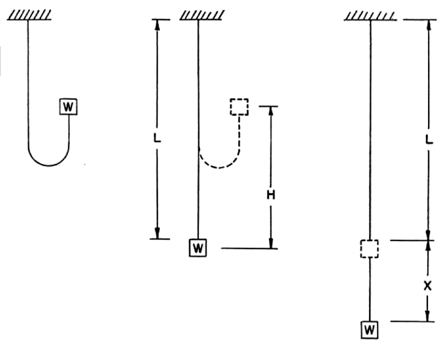


Figure 4 – Fall factor *Q(=f)* is the ratio between the fall metering and the open rope (leader-belayer distance) of maximum value 2. Adapted from Attaway (1996, p.6).

Table 1 – First published derivation known of the F_{MAX} formula, 1950, by Arnold Wexler, member of Sierra Club, California, replicating an essay of his own written in 1946 in the Club Bulletin (Wexler, 1950).

0	The F_{MAX} formula by Wexler (1950), published in the American Alpine Journal. According to Wexler, at the end of the fall, the climber has v=0. At that moment, all forces are in balance. As the climber had potential energy, that energy is now loaded in the tensioned rope. Potential energy = energy loaded in spring **W(H + x) = (1/2) Px** **[Eq.T1a]** P = Maximum force, x = rope elongation	0	
1	$P = \frac{K}{L} \cdot x$ **[Eq.T1b]**	2	Replacing Eq.T1b in Eq.T1a $W(H + x) = \frac{1}{2} \cdot \frac{K}{L} \cdot x^2$
3	Expanding equation $WH + Wx = \frac{1}{2} \cdot \frac{K}{L} \cdot x^2$	4	Rearranging terms $\frac{1}{2} \cdot \frac{K}{L} \cdot x^2 - Wx - WH = 0$
5	Grouping terms with "x": $x^2 - \frac{2WLx}{K} - \frac{2WHL}{K} = 0$	6	Solving quadratic equation for 'x': $x = \frac{WL}{K} + \frac{WL}{K} \cdot \sqrt{1 + \frac{2KH}{WL}}$
7	From Hooke's Law, $P = \frac{K}{L} \cdot x$	8	Then, $P = W + W\sqrt{1 + \frac{2KH}{WL}}$ **[Eq.2-1]**

Wexler warned that his formula did not consider the effect of dynamic belay (DB), nor that of rope slackening. He also disregarded the frictional force at the last clip, which may be explained by his concern with the strength of the ropes then, rather than the anchorage. From any point of view, it was a remarkable work, for being the pioneer in

5 The effective amount of rope involved in the fall dynamics, i.e. the rope meterage between the leader and his belayer.

unveiling the fall factor and considering factors discussed today, 68-years later, such as DB and dry friction, even speculating that dry friction could cause a fall factor (**f**) that reached or exceeded 2 (see below), a known fact by current researchers (e.g. Leuthäusser, 2016a, p.9; Lima-e-Silva, 2019c, in press).

However, in real life, under normal conditions, a factor-2 fall is not possible. A set of factors conspire against that fall factor and F_{MAX}, attenuating its magnitude: slackness of the rope, dry friction that draws energy from the system, damping in the braking apparatus or system, tapes, harness of the belayer, tightening the knots, dynamic belay, cushioning of the falling leader's body, as well as in his clothes and harness. These are the reasons why the UIAA applies a fall factor of 1.77 (UIAA, 2016) taken as the maximum possible value under real conditions.

2.2 Attaway (1996): Rope System Analysis

Stephen W. Attaway (1996) first does an F_{MAX} analysis with focus on elongation, coming to different forms of the equation. He uses the Greek letter δ to represent the elongation, instead of the common "x" and "y", and the index "st" to qualify the static displacement. He uses the capital letter "**K**" for the large modulus and assumes that this constant does not change when the spring undergoes a static or dynamic tension, to complete the Equation 2-2 format (Figure 5). This format has the disadvantage of making F_{MAX} to appear dependent of height **h**, which we know it is not, since the static elongation is directly dependent on the length of the rope considered; the longer the rope length, the greater δ, keeping the relation between **h** and **L** constant.

Further on in his text, he derives the elongation as a function of the rope modulus and replaces it, causing the formula to return to its best-known expression (Figure 5, Eq.2-3). The disadvantage of Attaway's paper for rock climbing in Brazil is that the author has been involved in rescue operations. His paper focuses on static ropes, and furthermore, those manufactured and in greater use in the USA, less used in Brazil. Static ropes are used by spelunkers and speleologists that have to climb up back abysses or cave entrances hard or even impossible to be climbed back. Climbers who have tried to go up on a rope hung down from above know how difficult it is to struggle against the elasticity that requires a lot of the effort to go up, making a long way up tiring.

$$\frac{F}{W} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} \quad \textbf{[Eq.2-2]}$$

$$\frac{F}{W} = 1 + \sqrt{1 + \frac{2hM}{WL}} \quad \textbf{[Eq.2-3]}$$

Figure 5 – Maximum impact force formulas (F_{MAX}) as in Attaway (1996). Firstly, Eq.2-2, with two problems, it appears to explicit a non-existence dependence of F_{MAX} with h, and assumes implicitly that the modulus is equal in the static and dynamic cases, which is not true; then, Eq.2-3 removes the dependency and enter data provided by the manufacturers. F=maximum force; W=weight of the climber; h=length of the fall; δ_{st}=static stretching; M=rope modulus; L=open rope length.

2.3 Jimenez and Freitas (1999): Study on Fixed Protections used in Brazil (Bolts) – JF

Jimenez e Freitas wrote a kind of technical report about an experiment with bolts, with no scientific methodology, either in the writing or in the experiment set up, resulting in an informal report. Though, it was included in this paper as part of the historical description in chapters 2 and 3, as part of the sensitivity analysis, because is one rare experiment, even not scientifically formatted, that is documented and published on the website of CEC (a climbing in Rio de Janeiro), in Portuguese, focusing on bolt resistance and F_{MAX}.

The document is so far the only reference in Portuguese found in the bibliographic survey for this paper that addresses a formulation for F_{MAX}. The authors have derived a formula by making hypotheses similar to those of other authors throughout time (e.g. from Wexler, 1950, to Leuthäusser, 2016a), but with different terminology and additional simplifications whose boundary conditions are not explored (Figure 6).

They present two equation formats of F_{MAX}. The format of Eq.2-4 (Figure 6) is said to be simplified, because it ignores on its derivation the elongation displacement, justified by assuming that "…*the elongation Δx due to the rope's elasticity is too small compared with the fall distance*". For the dynamic ropes "too small" is undefined and questionable, since the dynamic elongation of those ropes are about 35% on average, too big value to be negligible. The simplified one has a suppressed term based on the fact that even though it keeps producing not so bad approximations. Simplifications are always used in science and specifically Engineering, since the proper care is given, as to limit the scope, to point restrictions and evaluate the possible errors, inexistent aspects in the work.

$F=\sqrt{2K.(mg).\left(\frac{2H}{L}\right)}$ **[Eq.2-4]**	$F=P+\sqrt{2K.P.Q}$ **[Eq.2-5]**

Figure 6 – Eq.2-4 in JF (1999) for the simplified case. F=maximum impact force; m=climber's mass; g=gravity; H=length of the fall; L=open rope length; M=Modulus of the rope. Eq.2-5 in JF (1999) for the most complete case. Q(=f)=H/L; P=mg.

Also, the variables and tools of Physics are not always explicit. They used Hooke's Law (p. 5), but it is not quoted, and explanation for rope characteristics does not happen, as well; the elastic constant of the rope – on the paper represented by "**c**", instead of the classic "**k**" – also does not receive this denomination, but "constant of proportionality". The capital letter "**K**" is assigned to "**...***a constant that depends only on the rope used*...", which is the rope's modulus already mentioned in Footnote 6 (p.3/26), without making that meaning explicit.

Jimenez and Freitas ("JF") had the merit of pioneering in Brazil an experiment in search of answers regarding the resistance of the P-bolts used in Brazil, but it needs improvements. The work lacks scientific rigor in language and method, and can not be replicated or verified. One immediate finding is that they ignored the developments of previous researchers, which would have saved them time and made the work more robust.

Comparing the analyzes of other authors (Wexler, 1950; Leuthäusser, 1996a; Attaway, 1996) with the development of JF, it is easy to see that "**K**" (force dimension) is actually "**E•A**", the product of Young's modulus by its cross section.

The formula in Figure 6 (Eq.2-5) is put by the authors as the "*most complete*", for not disregarding, more correctly, the displacement during the elongation as in the deduction of Equation 2-4. This difference clearly appears when we plot the values of F_{MAX} for both equations (Figure 7).

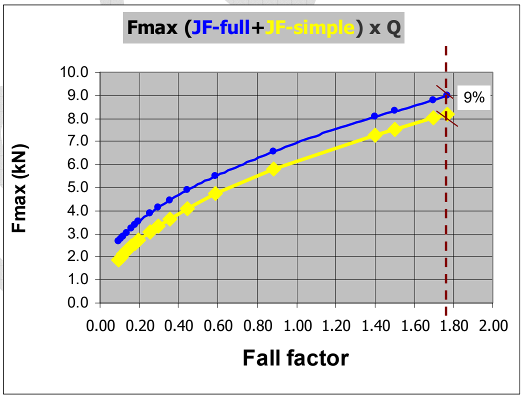


Figure 7 – **Plotting of simplified and complete JF models versus Q(=f). Note that the difference is relevant and that the simplified formula should be abandoned.**

Despite the non-scientific methods and some questionable conclusions, there are meritorious points of mention, discussed in Chapter 3.

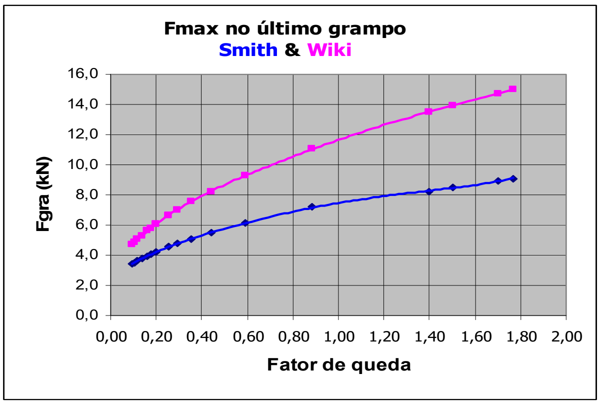
2.4 Smith (2016): Fall Force Calculator[11]

Physicist Casey Smith (2017) has created a calculator on a web page (junkfunnel.com/fallforce), "Fall Force Calculator". The calculation model is not visible, just a data entry screen (Figure 8b). After the data is provided in the window, a "Calculate" button is clicked on, and the routine behind calculates and displays the results on the browser screen, which include F_{MAX}, F_{BOLT} and F_{BEL} (maximum belaying force), and the impact factor on the climber (F_{MAX}/mg, "g-factor", acceleration of gravity supported by the climber on braking). A non-explicit model restricts the proposal from a scientific point of view, but by conveniently varying the data, the results and the requested variables provide good clues.

The user must enter: climber's mass ("weight"), static elongation of rope (provided by manufacturers), open rope length, distance to last clip, and friction factor on to last carabiner. The static elongation request (for m=80kg) indicates that the rope modulus in the calculation model should be estimated through static behaviour. A positive novelty is the consideration of friction on to the last carabiner, a relevant and always present factor in a real climb and generally disregarded in the formulations of many authors (e.g. Beal, 2004; Wexler, 1950). Making estimates with Smith's calculator, and comparing results with Wikipedia model, we obtain the chart of Figure 8a.

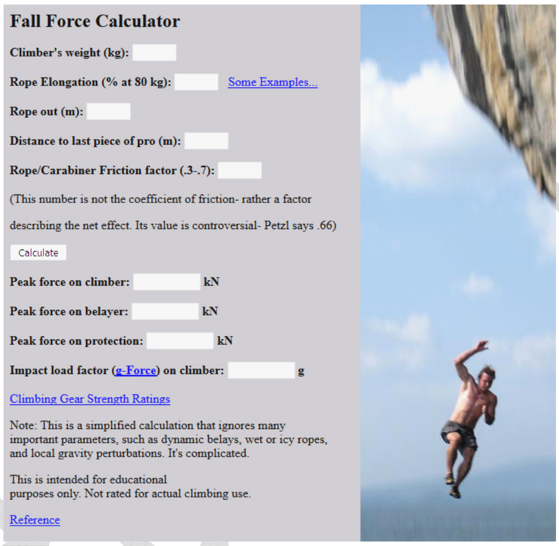
Although the internal model is not presented, in a personal communication Casey Smith kindly sent his calculation model, confirming the initial deductions from this study (Smith, 2017b). The Smith's Calculator routine uses

Equation 1 in Figure 8a to make the calculations, similar to that of Wexler (1950). However, it calculates the rope modulus using Hooke's Law with the static elongation informed by the user, and replacing it by the modulus of the rope (**F=K(x/L)**; so **K=mg/e**). It is important to remember that the climbing rope is not a spring, it is way more complex than that, and deriving the elastic constant from the static stretching does not explain its behaviour during an impact. That explains the difference between the results of the Wikipedia formula (based on dynamic elongation) and the values obtained from Eq.2-1 using **K** as derived by Smith, based on static elongation (Figure 8a).



$$P = W + W\sqrt{1 + \frac{2kH}{WL}}$$

(a) Eq.2-1



(b)

Figure 8 – **(a) Chart comparing F_{MAX} from Smith's Calculator (blue) and Wiki's (pink). Other values for the chart: P=Maximum impact force; W=climber's weight; k=rope modulus; H=fall height, L=open rope. (b) Image of Smith's Calculator webpage. The user fills out the data and the page promptly shows results (Url:junkfunnel.com/fallforce/, access on [31/March/2017]).**

Anyway, we should applause two Smith's Calculator attributes, the practicality of use, direct and fast in obtaining results, and the consideration of F_{BOLT}, the maximum force onto the last bolt. If imperfect in modelling, the presentation and availability of the results leads the user to consider the issue as an important part of the safety system that keeps him(her) alive, and stimulates the study of the theme, as it did with this author.

2.5 Wikipedia (2017): *Fall factor* Leuthäusser (2016a): *The omnipresent impact force formula for a climbing rope*

Both these references appear together because the content of Wikipedia (2017) has Leuthäusser (2016a) in the bibliographic references, so they seem to have similar rationale, although the two sources present some different aspects, which are relevant to this analysis (Figure 9). Wikipedia is not a source of scientific information, because it uses an open architecture of collaboration, receiving contributions from around the world and contains unverified, scientifically validated terms. But it has imposed itself as a consecrated source of unrestricted access information. An assessment of the scientific journal Nature has surprisingly concluded that Wikipedia's error rate is only slightly higher than that of the also famous Encyclopaedia Britannica (Nature, 2005).

$F_{max} = mg + \sqrt{2mgEAf + m^2g^2}$ **(a) Eq.2-6**	$F_{max} = mg + \sqrt{(mg)^2 + 2mghk} = mg + \sqrt{(mg)^2 + 2mgEqf}$ **(b) Eq.2-7**

Figure 9 – **Eq.2-6 is F_{MAX} as it appears in Leuthäusser (2016a, p.3). Eq.2-7 as in Wikipedia (2017). It is trivial to see that they are the same formula as Eq.2-1. F_{MAX}=maximum force on climber; m=climber's mass; g=gravity; E=Young's modulus; A=q=rope's cross section; f=fall factor; k=rope's modulus.**

Critics point out that the comparison was not fair, because it was based primarily on scientific inputs, and since these in Wikipedia are usually elaborated by experts, it had an advantage. The comparison was between the English editions. Despite the acknowledged weakness of Wikipedia, with its thousands of unverified entries, and how many errors potentially occur therein, in the case of concepts about climbing physics we have one example of scientific concepts, and one we can especially check.

$$F_{MAX} = mg + \sqrt{(mg)^2 + F_0(F_0 - 2m_0g)\cdot\frac{m}{m_0}\cdot\frac{f}{f_0}} \quad \textbf{[Eq.2-8]}$$

Figure 10 – Formula of the maximum force as it appeared in Wikipedia (2017, *Fall factor*) before correction. F_{MAX}=maximum force; m =mass of the climber; g =gravity acceleration; F_0=maximum rope force in test; m_0=test mass; f =fall factor; f_0=test fall factor. The subscript "0" (zero) indicates data from UIAA test. m_0=80kg, f_0 = 1.77; F_0 is published by the manufacturer for each rope.

The concept of Fall Factor, in Wikipedia, brings a calculation model of F_{MAX} ("Wiki") with the direct references of Leuthäusser (2012, 2016b, 2016c) and Goldstone (2006), the latter using the same equation published by Wexler (Figure 3). In this case, Wikipedia was considered for assessment, although I have found a small deficiency for specific situations. In this entry, Wikipedia first used to present the format in Figure 9 (Eq.2-6), equivalent to that of Leuthäusser (2016a; Figure 9a), and then presented an interesting second format (Figure 10), in which it had replaced the rope modulus, usually unknown, by variables available to the public on maker's website. The deficiency found is that the second format should show a warning saying that it is valid only for places with a gravity equal to the UIAA's test gravity. If applied to any situation with a different gravity acceleration, Eq.2-8 fails.

Table 2 – Derivation of expression for K in order to eliminate itself in the formula of F_{MAX}.

S1	$F_0 = m_0.g_0 + \sqrt{(m_0.g_0)^2 + 2.m_o.g_0.K.f_0}$	**S2**	$F_0 - m_0.g_0 = \sqrt{(m_0.g_0)^2 + 2.m_o.g_0.K.f_0}$
S3	$(F_0 - m_0.g_0)^2 = (m_0.g_0)^2 + 2.m_o.g_0.f_0.K$	**S4**	$F_0^2 - 2.F_0.m_0.g_0 + (m_0.g_0)^2 = (m_0.g_0)^2 + 2.m_o.g_0.f_0.K$
S5	$F_0^2 - 2.F_0.m_0.g_0 = 2.m_o.g_0.f_0.K$	**S6**	$K = \frac{F_0^2 - 2.F_0.m_0.g_0}{2.m_o.g_0.f_0}$
S7	$K = \frac{F_0(F_0 - 2.m_0.g_0)}{2.m_o.g_0.f_0}$	**S8**	$F_{max} = mg + \sqrt{(m.)^2 + 2.mg.K.f}$
S9	$F_{max} = mg + \sqrt{(mg)^2 + 2.m.g.\frac{F_0(F_0 - 2.m_0.g)}{2.m_o.g.f_0}.f}$	**S10**	$F_{max} = mg + \sqrt{(mg)^2 + 2.m.g.\frac{F_0(F_0 - 2.m_0.g)}{2.m_o.g.f_0}.f}$
S11	$F_{max} = mg + \sqrt{(mg)^2 + 2.m.\frac{F_0(F_0 - 2.m_0.g)}{2.m_o.f_0}.f}$	**S12**	**How it was on Wikipedia:** $F_{max} = mg + \sqrt{(mg)^2 + F_0(F_0 - 2m_0g)\cdot\frac{m}{m_0}\cdot\frac{f}{f_0}}$
C9	$F_{max} = mg + \sqrt{(mg)^2 + 2.m.g.\frac{F_0(F_0 - 2.m_0.g_0)}{2.m_0.g_0.f_0}.f}$	**C10**	**How it is now corrected (Eq.2-10):** $F_{max} = mg + \sqrt{(mg)^2 + F_0(F_0 - 2.m_0.g_0).\frac{g}{g_0}\cdot\frac{m}{m_o}.\frac{f}{f_0}}$

For example, one obtain a F_{MAX} of 9kN (using UIAA data) even if **g=0**! In a zero gravity field, F_{MAX} can only be zero. We find out a possibility why that happened if we follow the deduction of the equation to calculate **K** (Table 2, problem in step S10). To eliminate **K** from the equation, F_{MAX} equation was applied to the UIAA test itself, and the expression found was took back into F_{MAX} general formula (Table 2, steps S1 a S10).

The UIAA test generate data about the ropes, and all commercial ropes need to compliance with UIAA standards to get the certification. The wiki format use the test data to eliminate rope modulus **K** and provide a formula that allows to enter arbitrary values of climber mass, fall factor, and F_{MAX} as published by makers.

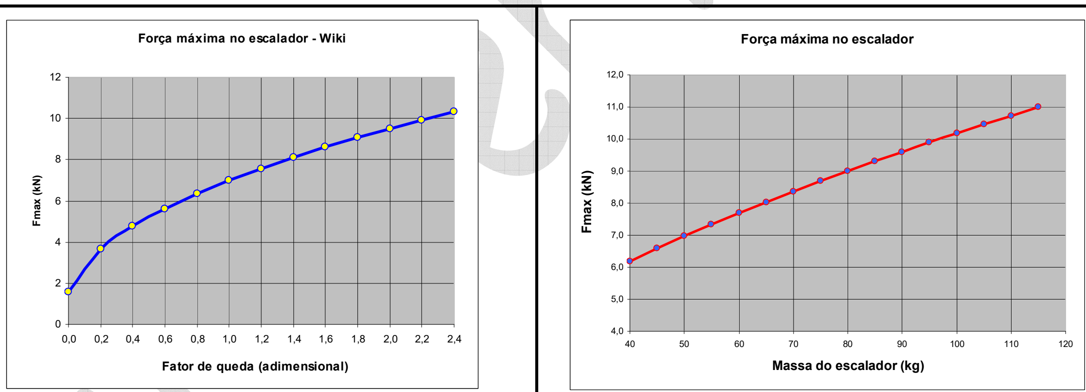
The "disappearing" of **g** happened because there was in our opinion an inappropriate simplification of the **g** on top of the fraction inside the root (coming from potential energy) with the **g** down the fraction (coming from UIAA test) in step S10. If the fall being analysed were in a gravity field other than Earth's surface value, it would not be valid. This author discussed the problem with two other researchers, Casey Smith, former MIT Physicist (Smith, 2017b) and Dave Reeve, from Australian Climbing Association (Reeve, 2019). We agreed that former wiki equation (Eq.2-8) was inappropriate, and Smith proposed the solution that corrected the problem[6] (Eq. 7).

Refer to Equation 2-10 (C10, Table 2). The subscript zero belongs to the variables from UIAA test. Keeping both **g**'s and indicating their origin, the same way that was done for **f** and **m**, the equation can be applied for gravity fields other than $9{,}807 m/s^2$. That is the case, for instance, when analysing the maximum impact force on non vertical slopes, predominant in Brazil's geology, where gravity acceleration is different.

3.0 CHART SENSITIVITY ANALYSIS

3.1 Wexler's Formula in Wikipedia Format ('fall factor')

The climbing ropes come over time becoming more and more elastic, absorbing more energy from the fall and reducing F_{MAX} imposed on the climber and the safety system. This tendency is observed in the continuous reduction of F_{MAX} value at the UIAA test of the ropes year by year, progressively distancing of the maximum limit of the UIAA (12kN). This state of art in the rope making contrasts sharply with the ropes of Wexler's time, whose concern was justifiably the strength of the ropes and not the anchors as today.



(a) (b)

Figure 11 – (a) Variation of F_{MAX} with fall factor, according to Eq. 6. (b) Variation of F_{MAX} with climber mass, according to Eq. 6. Fixed parameters: m=80kg; m_0=80kg; g=9,81m/s^2; F_0=9kN; m_0=80kg; f_0=1,77.

The F_{MAX} formula as it appears on Wikipedia (formula "Wiki") is physically similar to Wexler's, only the symbology has changed. The difference is that Wikipedia presents a second alternative format, in which the rope modulus (**K**), an usually unknown quantity, was replaced by an expression containing known values produced by UIAA ropes tests. This formula is more flexible, since it allows to evaluate F_{MAX} according to the mass of the climber, the falling factor and, after the correction done during this study (See Section 2.5, Eq.7), in situations with arbitrary gravitational acceleration. The latter will allow, in ongoing studies, to estimate and compare F_{MAX} values on slopes. In addition, the formula allows the user to calculate F_{MAX} for any other standard test. Thus, it gains independence from the UIAA standard and another variable (**g**) for which values can be arbitrated. Figures 11a,b show the F_{MAX} as a function of climber's mass (**m**) and fall factor (**f**).

Despite this flexibility, it is important to remember that the values of F_{MAX} should be interpreted with restrictions, because Eq. 8 disregards other important intervening factors, such as last carabiner friction (LCF), dynamic belay

[6] See on Wikipedia the term *fall factor*, where the changed equation now allows for arbitrary gravity accelerations.

(DB), dry friction and the slack in the rope. The first graphical analysis of F_{MAX} is its variation with the fall factor (**f**), which presents an increasing behaviour and, from f=0.5, practically linear (Figure 11a). The maximum force used in the calculation was 9 kN, because it is a limit value for the vast majority of ropes currently marketed.

The chart of F_{MAX} vs. climber mass shows a behaviour similar to F_{MAX} vs. fall factor (Figures 12a,b), which is expected, considering that both variables have strong influence in the forces. However, mass and fall factor affect them on different aspects. A greater fall factor keeps the amount of energy, but decreases the rope length to absorb it. A greater mass keeps the amount of rope, but increases the energy. In the end, they cause quite similar results. One can see that beyond a 40kg climber mass, F_{MAX} has a quite linear variation, looking very much as a straight line instead of a squared root function, as predicted by Jimenez e Freitas (1999, p.7), who said that F_{MAX} would be proportional to the square root of climber mass (See Section 3.4).

3.2 The formula of Jimenez and Freitas (1999) – JF

For this analysis, only the format said to be complete on the paper will be discussed (Jimenez & Freitas, 1999). It has a suppressed term with the justification that it stands as a good approximation. Figure 12 repeats formulas of Eq. 4 and Eq. 3b for comparing. Even in this form, JF suppress the term of weight within the radical, under the argument that "...*in practice the term 2.KQ/P is between 10 and 100*...", and therefore much larger than the term of the square of the explicit weight in Eq. 4.

$$F_{max} = mg + \sqrt{2mgEAf + m^2g^2}$$

(a) [Eq.3-1]

$$F = P + \sqrt{2K.P.Q}$$

(b) [Eq.3-2]

Figure 12 – (a) F_{MAX} formula in Wikipedia (2017, Fall factor). m=mass of the climber; g=gravity; E=Young's modulus; A=cross-sectional area of the rope; f=fall factor. (b) F_{MAX} formula according to Jimenez & Freitas (1999), put as a "more complete format". P=mg; K=EA; Q=f.

Let's define "*in practice*". Assuming that it means "in the vast majority of cases", it embeds a mistake. The **K** value ranges from 8.0kN to 30.0 kN[7], and that of **f** (fall factor) from 1.77 to 0.00, remaining below 0.4 by more than 90% of a 40m pitch with 4.0m between anchors (Lima-e-Silva, 2017c, in press). With these values, the term **2KQ/P** ranges from 5 to 140. To justify the simplification of the formula, any upward error at the upper limit does not matter, because it further justifies the simplification. The relevant imprecision is the lower one, since for values of **f** below 0.4 the despised error (1/5) reaches 20%, a variation that should not be neglected.

Figure 12 compares Equations 3-1 and 3-2, the first one being similar to other authors', from Wexler (1950) to Leuthäusser (2016a), and the second one is Jimenez and Freitas' equation (in the charts "JF"), with their simplification. Figure 13 shows visually the differences between the equations, varying F_{MAX} with **Q** and **P**. The differences between JF model (Eqs.8-9) against Smith's and Wikipedia's models can be clearly seen on the charts, the latter ones using static and dynamic elongation modulus K correspondingly.

It can be seen that by using the formula of JF with a static **K** (derived from static elongation), F_{MAX}[JF] is significantly below even the values of F_{MAX}[Smith] using that same **K**, resulting in an error of no negligible magnitude (Figure 13a). Even considering that static **K** is not correct for this kind of calculation, since we are interested in the dynamic phenomenon, the comparison brings to light a failure of the JF model in explaining part of the phenomenon. When we use dynamic **K**, F_{MAX}[JF] is about 10% below F_{MAX}[wiki].

Other differences between the equations can be pointed out. For instance, Jimenez and Freitas infer that "*This force is proportional to the square root of the weight of the guide*", while others authors show that F_{MAX} is proximally proportional to the weight. Disagreements stem from the same problem, oversimplification in the derivation of the formula.

Figures 13 show two graphical analyzes comparing the JF formula with the Wiki one. It is interesting to note that, although the simplifications in JF model (Figure 6) are conceptually wrong, the final result, if we apply an

[7] From a value of ***K*** derived from static elongation (9.2%) of a low modulus rope (e.g. Edelweiss Geos 10.5) up to a value derived from the dynamic elongation (39%) of a high modulus rope (e.g. Mammut 9.8 Transformer).

estimated **K** for dynamic impact (K=24.100N; Beal, 2004, p.2) and assume F_{MAX}=9kN, presents a difference below 10% before the Wiki formula. However, if we use a **K** derived from a static elongation, the difference to **f=1.77** becomes large, about 100*(9-5.7)/9=36.7%. In any case, there is no comment or detail about **K** in their work.

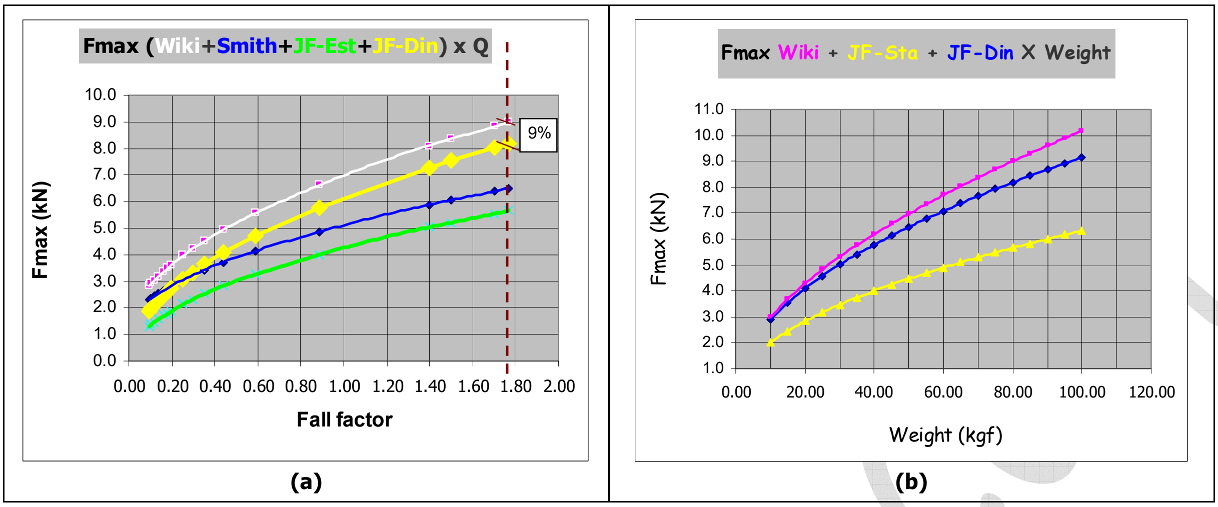


Figure 13 – (a) F_{MAX} curves vs. fall factor for Wiki, Smith, JF-Din (dynamic K) and JF-Sta (static K) equations. Note that JF-Sta stands even well below Smith's, which also uses K from static stretching. The difference is due to the oversimplifications in JF formula. (b) F_{MAX} curves vs. climber's weight. Again the large difference of the JF curves is due to use K from the static and dynamic elongations. Fixed parameters K_d=24100 (=E•A in Eq.2); K_e=11538 (6.8%); m=80kg; g=9.807m/s^2; 2H/L=1.77.

4. THE EXPERIMENTAL MODEL 1 – EM1

4.1 Introduction

The literature review showed that climbing physics has been well known since mid-20th century and continues to be improved with recently published papers. The interference of these two factors, last carabiner friction (LCF) and dynamic belay (DB) are also known. However, the two factors are not explicitly accounted for or considered in the many forms of the classical equation. It is agreed that LCF is always present in a real situation, as well as the fact that any reasonable belay is intrinsically dynamic and has a strong impact attenuating F_{MAX}.

The Smith Calculator (Section 2.4) is the only calculation model of F_{MAX} found in the literature that considers the LCF. It is also the only one that provides a practical way to obtain fast results, and which allows accounting not only F_{MAX} but also F_{BOLT} and G force on the climber. On the other hand, the equation of the model is not visible to visitors of the website, although Smith has sent it promptly to this author.

The proposal of EM1, besides the basic issue of a large gap of scientific papers in Portuguese on climbing Physics, is to bring a model in which the user can take into account all the important factors, and to be able to arbitrate values for the variables in game adapting the calculation to specific conditions that are useful to him.

4.2 The Last Carabiner Friction (LCF)

- **The Last Clipping**

The friction between rope and last carabiner (LCF) clipped is the dry friction part to be taken into account. It is on this carabiner that F_{MAX} expresses itself in its entirety, and where the true maximum force occurs, in the sense of the greatest force at play, because the two forces are added by pulling the bolt down, the one of the fall, on leader's side, and the resistance, on belayer's side. Counterbalancing the two, the Newtonian reaction of the anchorage, pointing up, therefore with an equal and opposite resistance (Figure 14). The friction force (F_{FRIC}) in Figure 14 is a schematic and non realistic representation of the net friction of F_{MAX} pulling on one side (and pressing the rope against the carabiner) and the friction force resisting to that tension (a force inside the system).

That resistance is called friction factor, not coefficient, because it includes the net result of the interaction rope/carabiner, which includes the complex friction force between rope and carabiner (carabiner shape is not simple), the bending force present (see below), and what else comes into play. The dotted vectors shown are schematic representations of the internal forces, so should not be operated with the external ones. It was used the symbol **M**, so it won't be mixed with a real friction coefficient, but the net result of that interaction.

The friction influence on F_{BOLT} is relevant. A mental exercise would help to understand F_{FRIC} role. If friction rope-carabiner were zero, it would behave as a perfect pulley with zero friction. The force on the fall side would be completely transferred to the belay side, and $F_{BEL}=F_{MAX}$. The anchor would have to resist 2•F_{MAX}, one on each side[8].

What happens is that the LCF consumes part of the energy, transforming it into heat. So, there is less energy to be resisted by F_{BEL}, reducing the sum of $F_{MAX}+F_{BEL}$, and so F_{BOLT} (the force that the anchor needs to withstand). For practical purposes, and considering that Figure 14 is a reasonable representation, this arrangement provides numerical explanations and results. Figure 14 provides these considerations graphically and mathematically. Conceptually:

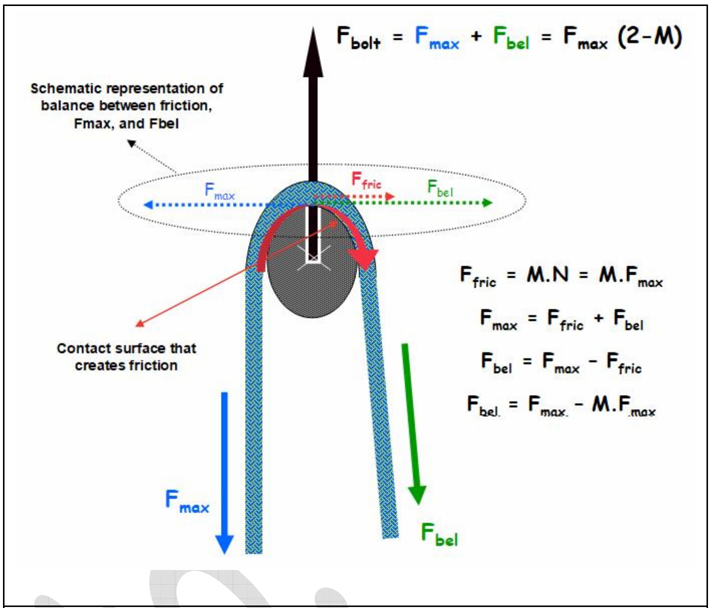


Figure 14 – Simplified scheme of forces on the last clip. The net friction result of rope-carabiner contact ("M") reduces the force onto the anchor (F_{bolt}). With zero friction, the force on the belay side (F_{bel}) would equal F_{MAX} (fall side), and $F_{bolt} = 2F_{MAX}$. With infinite friction, F_{bel} would not exist and $F_{bolt} = F_{MAX}$. Note that F_{FRIC} is an internal force, its depiction is just schematic.

Hypothetical carabiner without friction:

$$\mathbf{F_{BOLT} = 2 \bullet F_{MAX}}$$

Real carabiner with friction:

$$\mathbf{F_{BOLT} = 2F_{MAX} - M.F_{MAX}}$$
$$\mathbf{F_{BOLT} = F_{MAX}\,(2-M)}$$

A typical value for **M** is 0.34 (Petzl, 2005, *apud* Smith, 2017a,b), which causes a F_{BOLT} reduction of about 17%. But that's not the whole story yet.

- **The effective rope modulus**

Back to our hypothetical pulley-carabiner with zero friction, all the energy of the fall would be transmitted to the whole open rope, and all its extension would be elongated by F_{MAX}. However, considering the more realistic situation of the LCF[9], part of F_{MAX} is no longer transmitted to the belaying side. With less tension on that side, the rope lengthens less than on the side of the fall. For the falling climber, everything happens as if the rope becomes a little stiffer, with less stretching, as if the rope's modulus had increased. Therefore, to consider the entire open rope in the calculation, we must assume an effective rope modulus greater than the nominal one. Larger modulus means larger F_{MAX} (see Equation 2), which results in a reduction of attenuation caused by friction. Mathematically:

$\mathbf{F'_{BOLT} = F'max + F'_{BEL}}$ – F'max is the F_{MAX} that will occur considering the change in the modulus
$\mathbf{F'max = F_{MAX} + \Delta F_{MAX}}$ –Δ F_{MAX} is the increase due to the increase of the rope modulus;
$\mathbf{F'_{BEL} = F'max - M.F'max}$ – F'_{BEL} is the new F_{BEL} that conforms to F'max;

$$\mathbf{F'_{BOLT} = (F_{MAX} + \Delta F_{MAX}) + [(F_{MAX} + \Delta F_{MAX}) - M.(F_{MAX} + \Delta F_{MAX})]}$$
$$\mathbf{F'_{BOLT} = F_{MAX}(2-M) + \Delta F_{MAX}(2 - M)}$$

Where F'_{BOLT} is the maximum force on the last anchor considering the apparent increase in rope modulus. To know if we are overestimating the attenuation (the 17% from the above example) in a significant way by neglecting the increase in the effective modulus of the rope, we need to assess the magnitude of the term $\mathbf{\Delta F_{MAX}(2 - M)}$, the "attenuation of the attenuation" toward $\mathbf{F_{MAX}(2-M)}$. In reality, there is a simple way to check. If we can estimate

[8] The described case is the worst one, when the forces are parallel; in true life bolts are hardly in a perfect vertical line, and any deviate from vertical would reduce F_{BOLT}. For instance, if F_{MAX} and F_{BEL} were at 45 degrees with the vertical line, F_{BOLT} intensity would be 1,41•F_{MAX}, not 2•F_{MAX}, a 30% reduction due to the angle. In the limit, if the angle between them were 180 degrees, F_{GRA} wouldn't exist at all.

[9] At this point, assuming there are no other frictions along the way; I'll come back to this further in the paper.

the variation of the rope modulus as perceived by the climber, it would suffice to use the formula of F_{MAX} with the new **K**[10] and calculate F_{MAX} with the nominal **K** and with the apparent **K'**. It is quite clear that the difference depends on the relation (fraction) between the rope length on the belayer's side and on the fall side; for example, if the rope fraction on the belayer's side were zero, the nominal **K** would be equal to the apparent **K.**

A simple way is to evaluate two limits for this fraction: [i] if too small, we have the situation of a quasi fall factor 2 (**Q2**); and [ii] if close to 50%, a quasi fall factor 1 (**Q1**). Both results, as we shall see, are sufficient to figure out whether this change in the modulus is significant or not. In the first case, the fraction of the open rope on the belaying side would be about 10%, and we can expect a variation of **K** equally small. Since F_{MAX} depends on the square root of **K**, F_{MAX} would have an even smaller variation (K>1). In the second case, there is about half of the rope on each side, and the variation of **K** must be more significant. But does that mean a higher risk?

We may start from the basic definition of the stretches by writing:

$$\mathbf{e_{Total} = e_F + e_B} \quad \textbf{[Eq.4-1]}$$

where $\mathbf{e_{Total}}$ is the total elongation of the open rope, $\mathbf{e}_F$ the elongation of the fall side, and $\mathbf{e_B}$ the elongation of the belayed side. The objective is an expression for the variation of $\mathbf{K_{EF}}$ (effective K) felt by the climber as a function of the rope fraction on the belayed side. By Hooke's Law:

$\mathbf{F = \mathit{k} \bullet e}$ F=force, *k*= elastic constant, e=elongation
$\mathbf{\mathit{k} = K / L}$ K=rope modulus, L=rope length
So $\mathbf{e = (F \bullet L) / K}$ **[Eq.4-2]**

Substituting Equation 10 into the corresponding terms in Equation 9:

$$\frac{F\max\bullet(L1+L2)}{Kef}=\frac{F\max.L1}{K}+\frac{Fbel.L2}{K}$$

Substituting $\mathbf{F_{BEL}}$ by $\mathbf{F_{MAX}(1-\mu)}$, **L2** by a fraction of **L1**, and simplifying, we arrive at:

$$K_{ef}=K\bullet\left[\frac{1+T}{1+T(1-\mu)}\right] \quad \textbf{[Eq.4-4]}$$

Where **T=L2/L1, L2** is the rope length on the belaying side – from the last carabiner to the belay – and **L1** is the fall side meterage. Note that this formula applies to the problem as defined herein. Now we can assess the two situations, **Q1** and **Q2**. In a **Q2**, a typical fraction is **T=0.1**, because the climber falls, for example, from a height of 4.0m, thus an 8.0m fall, and the belayed side fraction has a magnitude order of 10%, or 0.40m. Assuming a typical value of **μ=0.34** (Petzl, 2005, *apud* Smith, 2017b), we calculated $\mathbf{K_{EF}=1.032 \bullet K}$, or 3.2% greater than the **K** of the rope. This variation in the F_{MAX} formula (see Eq.4) causes a F_{MAX} variation of +1.4%.

In a **Q1** fall**, T=0.5**, and $\mathbf{K_{EF}}$=$\mathbf{1.13 \bullet K}$, or 13% greater than **K**. Applying to F_{MAX}, but remembering to adjust **f**, which has changed (**f=1.0**) as well, we reach a variation of –13%, because the reduction of **f** overlaps with the increment of the effective rope modulus. What would be the reduction of F_{MAX} if we did not consider the variation of $\mathbf{K_{EF}}$? By making the F_{MAX} count with the original **K** of the rope, the reduction of F_{MAX} due to **f** would be 22.6%. Therefore, we see that for **Q1** the $\mathbf{K_{EF}}$ variation is significant in reducing the attenuation of F_{MAX} due to the reduction of **f**, but it does so for a F_{MAX} that is already going down, and tends to drop progressively in value as the climber ascends on the pitch. Figure 15 shows the phenomenon visually.

- **Rationale of effective K**

When it matters, at a time when there would potentially be a risk for anchors, in high **f** falls, the positive variation of **K** due to the increase in the apparent modulus of the rope is very small, and can be neglected. When the variation of **K** is large, in the falls of low **f**, the reduction of the fall factor overlaps with the increase of **K**, and the variation of F_{MAX} remains negative (Figure 15).

[10] Note for the symbols of the variables: the modulus of the rope is designated by **K** because it is the letter used by the authors in general for this concept. But the elastic constant of the rope is also called **k**, lower case, though. It may be confuse sometimes.

About the friction analysis, there is still a thing to look at. In our mental experiment, the opposite extreme is missing, i.e. what would occur if friction, instead of zero, was greater than $\mathbf{F_{MAX}}$ ($\mathbf{F_{FRIC}>F_{MAX}}$)? In that case, the belayer would not feel anything, $\mathbf{F_{BEL}}$ would not exist, and $\mathbf{F_{BOLT} = F_{MAX}}$. It would be the situation where $\mathbf{F_{BOLT}}$ would have its maximum reduction, if ever possible. In real life, $\mathbf{0 < F_{FRIC} < F_{MAX}}$, and $\mathbf{F_{BOLT}}$ is greater than $\mathbf{F_{MAX}}$.

The findings of Tanzman (2009) support those conclusions. His findings through a detailed derivation in the paper "*Incorporating Friction into the Standard Equation for Impact Force*[11]" of an adjusting model are clear, in which he derives the following equation:

$$f' = \frac{2f}{2+(f-2)\cdot M} \qquad \textbf{[Eq.4-5]}$$

Tanzman calculates F_{MAX} in two ways, with the classical form and its model of adjusting the rope modulus, which becomes in practice an adjustment in the fall factor, plotting the chart for two values of the friction factor, **M=0.33** and **M=0.20**. In the chart (Figure 16) one can see how the conclusions drawn from the simplified equation are also found in the sophisticated Tanzman model. One can see that the variation of **K**, translated in the article into an apparent variation of the fall factor, is also great when it has little influence on F_{MAX}, for low **f** values, and when it matters, in the region of high **f** values, the apparent modulus has little influence because the difference between the calculated apparent modulus and the nominal modulus of the rope tends to zero.

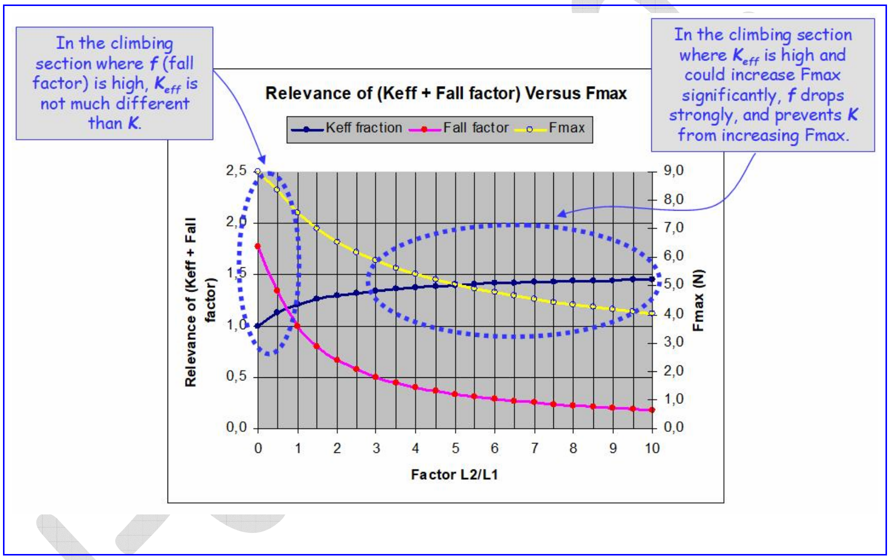


Figure 15 – Three curves on same chart in function of ratio L2/L1 (L1=belay side, L2=fall side). It can be seen that changes in K for evaluating F_{MAX} is irrelevant in a general sense, because it increases when fall factor is diving and drags F_{MAX} down anyway. Note that F_{MAX} curve had its values forced down to appear in same scale. It was done so the reader can see its behaviour while the fraction L2/L1 increases.

On page 4, beginning of Chapter 3, it appears that "*In a fall factor 2, the fall is directly on the belayer. The rope does not run through an anchorage above, so there is no friction,*" A fall "directly on the belayer", posed by the author without questioning, as a normal situation, is not right. There must be friction, for the rope is passing through a clipping at the base anchor, the other way is rare, with no clipping between climbers.

Besides that, there are two more flaws or gaps; he does not explain that factor 2 is not real, recognized by the UIAA itself and its standard of **f=1.77**, and seems to ignore that the carabiner-rope friction factor is a factor and not a coefficient of friction as understood in Physics, since it is represented by μ, symbol used for coefficient of friction. The friction between rope and carabiner has a net result, to which the author is referring, as he demonstrates in stating that "*This frictional force is conventionally assumed to be equal to 1/3* F_{MAX}." He refers to the value of 0.34 that was once published by Petzl, whose consequence can be interpreted as a counterforce of

[11] "Incorporando o Atrito na Equação Padrão da Força de Impacto".

about (1/3)•F_{MAX}. Denoting it with the Greek letter μ may lead the reader to think that the "μ" of Tanzman is a real coefficient of friction. This issue was explained at the beginning of Section 4.2. Finally, the friction factor embeds yet another relevant effect explained below.

Finally, it is important to mention that this "friction factor", when measured in a real situation, does not only incorporate an increase of resistance in the last clip due to the friction, but a fraction of it is due to the folding force. It is easy to see that there is such a force, although it is not mentioned in almost all texts, such as Attaway (1996; 1999). The only paper found that addresses this issue explicitly is Titt (2009), who takes this discussion in greater detail. Qualitatively, it is simple to realize its existence, it is enough to take, for instance, a rope with diameter of 11 mm and try to bend it in a curve of radius equal to or smaller than 11 mm.

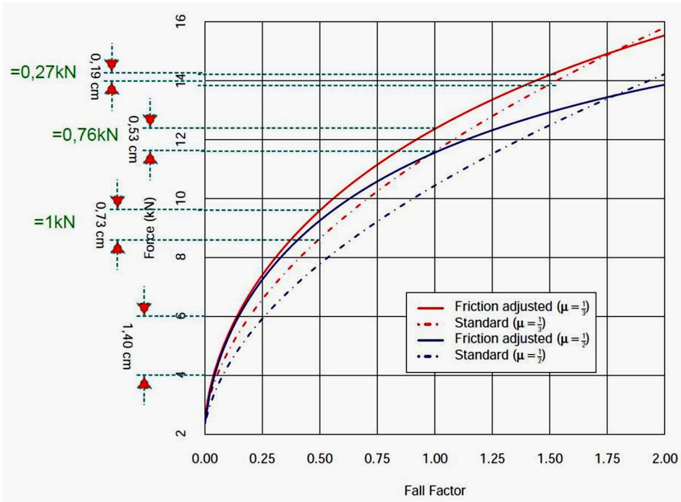


Figure 16 – The solid lines are corrected curves for the apparent variation of the rope modulus, in Tanzman model via adjusting fall factor. The dashed lines are the classic model results for the same friction factor. Correction is relevant when it does not cause impact on F_{MAX}, in the low fall factor range, just the same conclusion from the simplified model described (chart adapted from Tanzman, 2009).

If we try to make a rope pass through a pulley of this order of radius, even a zero friction pulley, we will have to exert an additional force, independent of the friction, which is the resistance of the rope in folding the inner part to curve, strangling the fibres, as opposed to the outside of the curve, which is forcing the extension of its fibres. It is interesting to note that the curve of the rope in the modern carabiners, much finer than the old ones, fits exactly in this situation. However, for all purposes, in cases where the friction factor in the carabiner has been measured, and not theoretically deduced, it embeds the effects of the folding force, and does not require further detailing, becoming an internal element of the friction factor, to some extent irrelevant to this analysis. But it is noteworthy that even if the friction in the curve were zero, the force applied on one side to make the rope move would be greater than the resistance force on the other side, resulting in an effect similar to that of a friction in the curve.

4.3 Dynamic Belay (DB)

- **Concept**

DB (dynamic belay) is a belay method that attenuates the magnitude of the forces (F_{MAX}, F_{BEL}, F_{BOLT}) involved in the case of a rope leader fall by maximizing the fraction of energy absorbed in the braking process under the belayer's control. It contrasts with the SB (static belay), in which, inadequately, an unprepared climber (perhaps focused on eliminating the danger as fast as possible) tries to interrupt the fall as abruptly as possible or that his lack of attention causes an abrupt braking.

According to Cragmont Climbing Club website (CCC, 2017), in a paper in the history section, and replicated on website touchstoneclimbing.com in 2014 (Touchstone, 2014), DB was invented in Indian Rock, in 1932, by Dick Leonard, a member of that club. The climbers jumped from a platform and gave each other a rope slack to make the falls softer. They used static ropes and protected themselves with padded clothing to prevent burns.

Steve Roper (CCC, 2017, p.1) tells the story and it is worth quoting a passage:

> *"Since safety was uppermost in everyone's mind, the RCS climbers, now numbering fifty-two, spent much of 1932 and 1933 learning to belay and rappel properly. They quickly discovered that the European shoulder belay – or its variant, the under-the-armpits belay – wasn't for them: it was a crude and even dangerous technique. So they invented the hip belay, in which the rope was placed around the lower waist, thus affording a more stable centre of gravity. The belayers also experimented with letting the rope slip slightly around the waist when bringing a falling leader to a stop. This "dynamic" belay eased the strain on the two humans and the rope itself, the weak link in the equipment chain."*

Since then, the climbing ropes have evolved immensely, and today absorb all the energy necessary to hold a fall. They are no longer the "weak link" cited by Roper (*op. cit.*). And it is in this matter that DB becomes important, because its ability to reduce F_{MAX} and F_{BEL}, the two forces to be compensated for by the resistance of the last bolt, is what will attenuate the resultant on the last bolt, and consequently the risk of failure.

But there are challenges to face. Although DB is an idea of universal knowledge, if you ask different climbers to describe it, you will get a variety of versions. In the meantime, a formal definition of UIAA has not yet emerged, which could establish a standardized reference and facilitate its dissemination and evolution. However, as we shall see, even a non-standardized DB is still dynamic, as it is virtually impossible, on a real climb, to avoid entirely the attenuation in the belaying. I want to argue that a strictly static safety in a real climb does not exist.

A SB would be the equivalent of a rope with no gaps, attached to the belayer's tip as a knot (previously tightened), secured to a solid, fixed point. An arrangement of this can be seen in the UIAA test (Figure 17). This way of securing the rope ensures that the belaying is static, and that the attenuation of the fall will be done basically by stretching the rope. This situation that can be created in a laboratory.

- **An illustrative simplification**

It is critical for a climber to understand the Physics that relate the time and braking distance of a fall to the magnitude of the forces at play. It is known that in order to dissipate a certain amount of non-reducible energy, the less time to dissipate it, the greater the intervening forces will have to be. An analogy will help. If someone wants to brake a speeding car in a short distance, he will have to exert more force on the brakes than if he has a greater distance to stop the car.

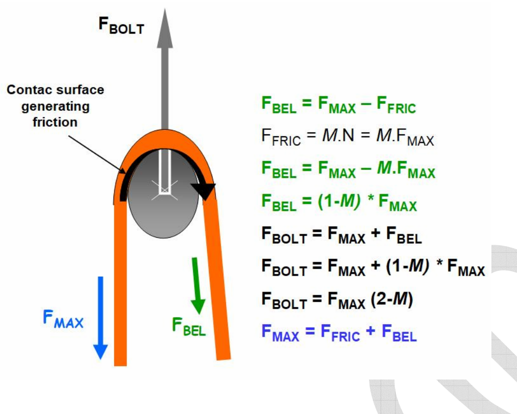


Figure 17 – The chart shows the energy as the product of the force by the distance, represented by the area A. We can consume the same energy E in two ways: using a greater force in a smaller distance (less time); or using a lesser force in a greater distance (more time);

This negative correlation between fall interruption speed (or distance) and the forces involved (F_{MAX}, F_{BEL}, F_{BOLT}) can be demonstrated with an illustrative simplification (Figure 17). The shorter the braking time, the shorter the running distance of the rope. Since the total energy does not change, a reduction in distance must be compensated by an increase in force.

Demonstrating mathematically how time and distance affect the attenuation capacity of DB over F_{MAX} for a real case requires deeper calculus, beyond the scope of this paper, but it is possible to visualize with a simplified mathematical model. Physics says that the energy required to perform the work of moving a solid body is the product force • displacement. For simplification, let's assume a constant force during the interval. This idea can be placed like this:

$$\mathbf{E = F \bullet D}$$

where **E** is the energy, **F** is the force, and **D** is the displacement from the beginning of the hypothetical "fall" to the complete stop. Since area **A** in Figure 17 represents all the energy needed to stop the fall, it is easy to note that if we increase the distance – which means increasing the braking time – we reduce the force required, and thus the forces acting on the system[12].

- **The real world**

As mentioned above, a strictly static safety in a real climb does not exist. Strictly, all belay is dynamic at some degree, because climbing action so demands. The movement of a falling climber's rope is never interrupted instantly, from the point of view of the speed the belayer can apply to his action. There are always extensions of the braking time and distance caused by several factors, most listed below, acting individually or together:

[i] Rope slacks across its open extent – because no climber moves with a tight rope – that in a fall will cause resistance by dynamic friction with the environment (rock, ribbons, carabiners, bolts), progressively increasing the falling climber toward the belayer as the system tensions;

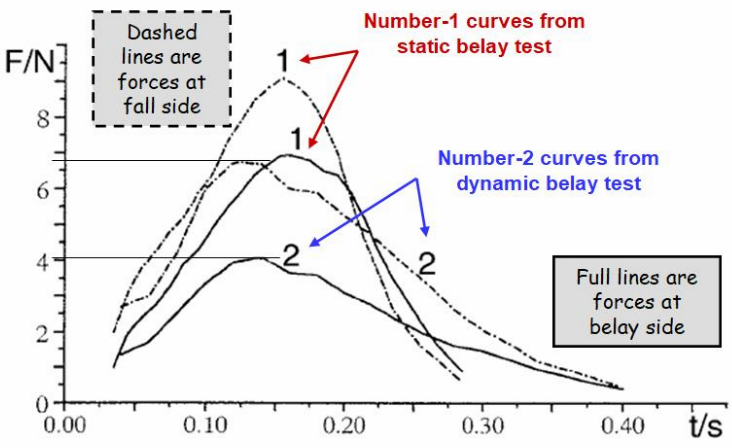
[12] In a real case the force is not constant, nor is the braking speed; however, the energy required to perform the braking work continues to be the area under the curve in the Force vs Distance chart; if the distance increases, a smaller force is required by the system to do the job.

[ii] Slackness between ATC[13] in harness and clip on the base bolt[14] and within the ATC itself;
[iii] Lifting of the portion of the belayer's weight after the slacks have been eliminated;
[v] Energy absorption for harness tensioning on the belayer's body and on the body itself;
[vi] Energy absorption by the fraction of the rope released by the belayer due to the time it takes between feeling the pull in the rope and effectively stopping it completely;
[vii] Time interval that leads to the roping knots on the two climbers being tightened to the limit.

In fact, virtually all cited factors intervene to lengthen the braking time and distance, and thereby attenuate F_{MAX}. In the 1970s, in the climbing clubs in Rio de Janeiro, there were some attempts of applying a supposed DB, of which one was heard to speak of, but nothing concrete was determined. Orally reported attempts account for daring belayers who had burned themselves a lot seeking the then unknown “dynamic belay”. Currently, even considering the absence of a standard, some published research and several expressed opinion of climbers allow us to sketch a basic arrangement.

- **The work of Czermin *et al.* (2007)**

In the scientific literature, the paper found that presents objective results on DB's ability to mitigate the forces at play was that of Czermin *et al.* (2007) (Figure 18). The chart of Figure 18a represents the forces measured in an experiment performed by those authors. If fall tests are done with and without DB, then the difference between the sum of the forces in the two cases represents the reduction of forces due to DB. The measurements show a 32.7% reduction for the sum of F_{MAX} and F_{BEL} (=F_{BOLT}, force on the bolt) with DB in relation to the same sum in the static and dynamic test. This means that the force that the bolt will have to resist becomes 32.7% less with the application of a dynamic belay, as performed in that test.



(a)

Curves 1 – Static belay (SB)
9.0 + 6.9 = 15.9 kN

Curves 2 – Dynamic belay (DB)
6.7 + 4.0 = 10.7 kN

Differences
Para F_{BOLT}: 10.7 – 15.9 = –5.2 kN
Para F_{MAX}: 6.7– 9.0 = –2.3 kN
Para F_{BEL}: 4.0 – 6.9 = – 2.9 kN

Percentage reduction of DB
F_{BOLT}: (–5.2/15.9) * 100 = –32.7%
F_{MAX}: (–2.3/9.0) * 100 = –25.6%
F_{BEL}: (–2.9/6.9) * 100 = –42.0%

(b)

Figure 18 – Chart adapted from Czermin *et al.* (2007). A fall test was done and the forces on fall and belayer sides were recorded. The 1s-curves are from static test, and 2s-curves from the dynamic one (a). The difference of the forces sum on both sides with/without DB is remarkable, as the numbers show, where F_{BOLT} is 32.7% smaller with DB. A strong proof that DB cannot be disregarded in the assessment of F_{MAX} and F_{BOLT} (b).

One might think that in the calculation just done, the friction was not taken into account. But note that friction is present in both cases, static and dynamic tests, therefore the difference between the sum of the forces in the static and dynamic case is actually accounting only for the attenuation caused by DB. The problem is that the authors do not explain in detail the test conditions, i.e. so we do not know what they exactly mean by “dynamic belay”. Given this uncertainty, the figure of 32.7% however seems reasonable to Leuthäusser (2017), who claims to have developed a mathematical model to evaluate a DB and achieved an impressive 40% attenuation for F_{BOLT}. Anyhow, until the test conditions of Czermin *et al.* are completely clarified, the value of 32.7% is not final, but still remains as an evidence of a strong damping effect.

[13] The "ATC" acronym, usually assigned to most commonly used devices for belaying (abseiling), can be replaced by any system with the same function thereof, to brake the rope progressively during a fall until its complete stop. It may be the HMS knot, or the former waist belay arrangement; any of them, if well done, does the job.
[14] Here "base" refers to the main anchorage to which the belayer is attached during the evolution process of the leader at that instant.

Figure 18b schematically shows the sum of the forces for the static and dynamic case of Czermin *et al*. I remind you that the static test arrangement for the fall of Figure 18a is artificial, based on the UIAA test arrangement, i.e. the secured tip is attached to a bar and the rope is rolled several times around the bar, thus creating an infinite friction simulation without any damping. That situation will never occur in a real climb.

- **How to standardize DB?**

The essence of the DB phenomenon is not complex, the problem lies in the impossibility of quantifying it in an analytical way, since there is no standard DB, and different DBs can lead to different attenuations, although, as already said and worth repeating, any reasonable DB arrangement will always cause significant attenuation. For an $\mathbf{F_{BOLT}}$ assessment that considers DB, one way to approach the problem would be to employ semi-empirical models, where users can adapt the variables to specific conditions and try to calculate values for different arrangements. However, assembling an empirical model would require carrying out a large number of tests, with variable changes (climbers' weight, mode of belaying, etc.), specifically for that purpose, which has not yet been done. Numerical analysis could then provide a set of responses that accumulate and evolve over time to form a clearer picture of how it impacts the forces at play.

I propose a form of DB, based on the reports of authors from other countries (Corrigan, 2016; Zanantoni, 2000; Titt, 2009) and on the experience of groups with whom I have been climbing for about 40 years, involved in falls of various natures. Nothing different than perhaps most climbers already do, perhaps a novelty for beginners who are not yet aware of the phenomenon and to some experienced ones not familiar with Physics. A reference facilitates the comparison, and encourages advances or variations of the technique, since the declivity of the route or pitch can lead to different stances. More important than the suggested posture are the principles on which it is based, which can be used in other postures. The suggested posture is not meant to be the "right" one, nor that different ones are "wrong", the suggested one is just a *reasonable* setup. Let us call it EDB, for Experimental Dynamic Belay. Figure 19 provides a schematic drawing.

- **Important points**

A posture of the belayer is "right", no matter what, if it meets a principle of stability. A fall should not unbalance the belayer because this would place at risk the breaking down system if he/she loses control of their hands. Any force that makes the body of the belayer spin, for instance, risks a clash of his/her hands with the environment, carabiners, ribbons, harness, rock, his/her own body, and opens the chance of either hand being hit. It is their hands that ensure a safe fall for the leader. The hand of the belayer that is primarily at the entrance of the ATC, must be protected so as not to be injured, pressed or subjected to any kind of shock that may cause it to open, because on it will depend the control over the climber's fall at the other end of the rope.

The EDB proposed herein in Figure 19, contrary to some definitions of a DB in websites, does not include active action by the belayer, taking into account two known rules from Safety and Reliability Engineering. The first one is that *the faster a human response is needed in an emergency, the greater the probability of failure*.

During a fall, events occur very quickly, in the order of seconds and tenths of a second, so actions during an emergency that directly impact the safety of the falling climber should not be based on practices that rely on a presupposed ability to be executed correctly in a very short time. The historical record tells us that, as time passes, and the event repeats itself, failures will occur. From this point of view, the less human action at the moment of a fall, the less probability of failure.

The second rule is that *the more independent elements a system has, the more chances that it will fail*. Thus, one of the ways to increase the reliability of any system is, whenever possible, to remove elements, reducing the failure probability. So, in the case of the DB, if we eliminate the event "*active action of the belayer*", we will be reducing the probability of failure. Referring to Figure 19, the blue circles are explained below.

Point 1: The central issue of DB is that it provides two fundamental advantages, space for the belayer to work and causes the down force to raise a significant part of the belayer's weight in case of a fall. The 45-degree angle is a good trade off between [i] workspace, [ii] belayer's comfort, [iii] leader vision, and [iv] counterweight to DB. Comfort in this context means a position that enables the belayer to focus on safety. Protecting the belayer is a basic requirement of rock climbing safety. The 45 degree angle requires, during a fall, that the force lifts, for an 80kg-climber, 80•sen(45) kg, plus the friction force at the base carabiner, which can be estimated at about 1/4 (45 degrees against 180 degrees of LCF), adding up to about 600N or 61kgf.

Point 2: The able arm (most people are right-handed) should face down, holding the rope entering the ATC, with the fingers down, to give greater force capacity resistance. The unfit arm (the left one, for most) must hold the rope coming out of the ATC, going towards the climber. These arm positions meet the equilibrium principle in a fall because they provide the belayer to control the input and output of the rope simultaneously.

Point 3: The hand of the ATC output is placed with the fingers (and the palm of the hand) facing the belayer, which harmonizes with the design of the human musculature and allows maximum extension of the unfit arm if it is pulled up. The ATC's input hand, the able arm, is placed in an opposing position, with the fingers facing down, the outside of the wrist facing the belayer; this position ensures maximum strength and extension of the able arm of the belayer.

Point 4: The ATC is attached to the belayer's harness shackle. This location ensures that the ATC always stays within a distance accessible to the belayer in case of any eventuality, besides the function of providing the permanent and continuous control of the device. The rope passing through the device during the leader's ascent should not be tight on the device, but have a relative slackness that allows the smooth and effortless movement of the belayer. This detail is part of DB. In the event of a fall, it will contribute to the dynamics of the braking, allowing for a greater distance and time in a braking process. As explained in Figure 17 and the items corresponding to it in this section, the length of time and braking distance contribute to increase the efficiency of the DB and thus reduce the forces in play.

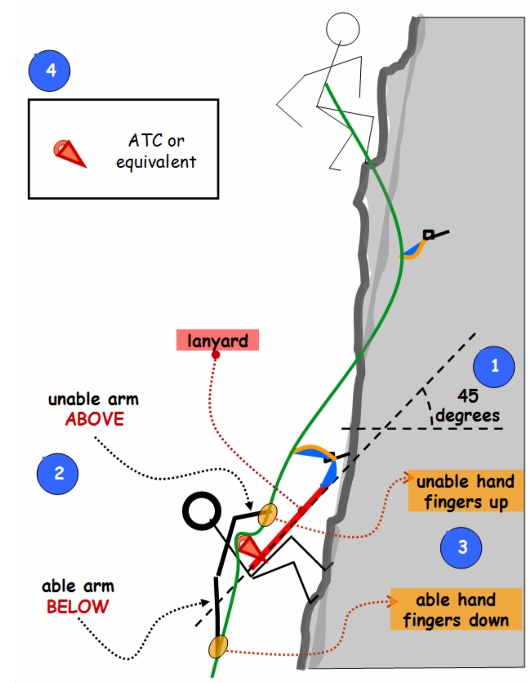


Figure 19 – Scheme showing an arrangement that produces an efficient DB, called here DSE, experimental dynamic belay. Usually, the able arm and hand are the right ones, and the unable are the left ones.

- **The risks**

DB embeds two risks. By providing a stretching of time and distance travelled by the rope with the purpose of increasing dynamic attenuation, it increases the chance for the falling climber to hit obstacles in his/her downward trajectory. Almost always, mitigating the forces at play is more beneficial than not mitigating them, considering that the biggest issue is to ensure lower forces on the system, and thus less chance of any failure. But special situations may point to a contrary necessity, that of reducing the distance of the fall rather than increasing it. The imminence of the climber hitting a sturdy obstacle, such as a rock point or a rocky plateau below, is a greater risk than stressing the safety system. Climbers, and especially the belayer, must judge what each situation requires, an intrinsic part of the sport, the continuous surveillance of situational risks.

The second risk is loss of control by the belayer. In a standard static safety situation, the belayer's participation is essentially passive, he does nothing but hold the rope so that it interrupts its movement as early as possible. In DB, he/she is expected to at least put self in a specific stance, which includes placing much of his/her weight on the PAS[15], and that the braking of the rope is progressive until the slackness between belayer and base carabiner, as well as inside the ATC are eliminated, but the degree of this progressiveness is controversial. Purposely letting the rope run a few meters is recommended by several authors, but in a traditional climb, the subject of this paper, the Risk Analysis knowledge says that an action requiring skill and having to be done in a short time adds unnecessary risk to loss of control of the fall with potentially serious consequences.

In the various discussion forums of rock climbers, there are widely held opinions about the belayer's behaviour, including actions lacking technical or scientific grounds, from picking up the rope hastily (which would cause an increase in the fall factor), to some theoretically safer ones such as letting the rope slip a few feet to reduce F_{MAX} (Zanantoni, 2000). The latter may be related to an issue already discussed by Lima-e-Silva (2017b), the fact that mountain morphology in the Northern Hemisphere is composed of a large number of vertical walls, concomitant

[15] Personal Anchor System.

with the more common use of mobile anchors, significantly less reliable than a bolt. In that case, the risk of losing control may be less than the risk of anchors failing.

In Brazil, the morphology is less vertical – routes generally have a positive declivity (slab) – and the anchorage is massively composed of 1/2" bolts, with zero records of catastrophic failure. The suggestion to let the rope run for a few meters embeds the risk of loss of control, already occurred in the 1970s in a climbing club (Fontenelle, 2017), because it is an action that requires speed of movement and ability to know how much rope to let go before of forcing the complete stoppage. It may be a procedure for few, with good training, good physical fitness and excellent and permanent attention in the leader, but from the point of view of public safety a risky idea.

This procedure also contributes to increase subjectivity and reduce systematization, which, in a rapidly growing population of climbers, will inevitably lead to increased number of accidents. The question of reliability of the belay system has already been addressed in the previous item ("Important Points"), and we can now add to the arguments against the belayer's active actions the high occurrence of positive walls and historically highly reliable anchors of the P-bolts. Putting together the arguments cited, we have a solid argument to refuse active action by the belayer. Better a not so attenuating DB but higher reliability.

The normally attentive and positioning belay stance, as schematized in Figure 19, coupled with the natural movement of the belayer who will take a few tenths of a second or more to immobilize the rope, to the energy consumption of lifting the body of the belayer by the force of the fall, and to other mitigating factors of friction and damping of the rest of the safety system are sufficient for an effective and efficient DB. Unfortunately, we will have to wait for the authors Czermin *et al.* (2007) to clarify which DB was used, or that new experiments are carried out, the forces measured, then further clarifications about the magnitude of DB attenuation may emerge.

4.4 The EM1 Equation

The EM1 sought to incorporate phenomena existing in the climb and absent in classical formulas in the literature, in order to make the model more realistic and so most realistic numbers for the impact forces of falls to be simulated on a computer. The phenomena addressed in this paper were the last carabiner friction (LCF) and the dynamic belay attenuation (DB). The formula is said special because it only covers free falls, and although it takes a significant step toward a more realistic assessment, it is still far from considering all intervening phenomena.

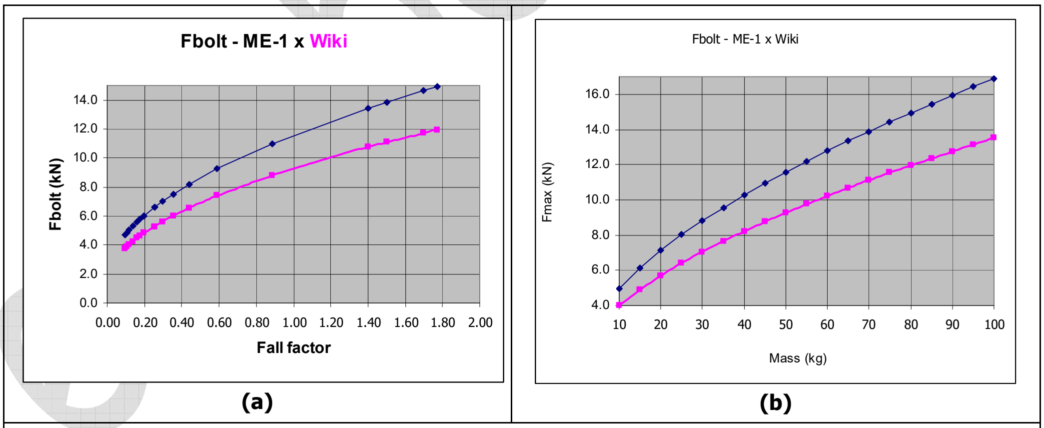
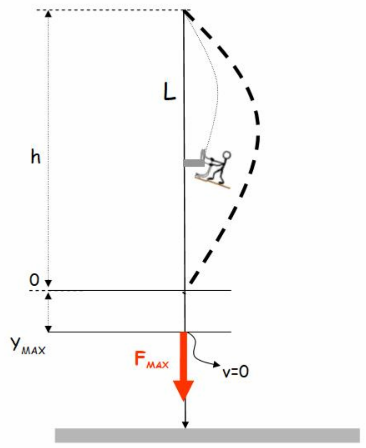


Figure 20 – (a) Model EM1 Vs. Wiki: F_{BOLT} as a function of fall factor. Other variables default. (b) Model EM1 Vs. Wiki: F_{BOLT} as a function of climber's mass. Default values: m=80kg; m_0=80kg; g=9,81m/s^2; f=1.77; f_0=1.77; α=90°; R_{DBmax}=0.80; M=0.30; F_0=9kN.

Figures 20a and 20b show two illustrative examples of the use of EM1 versus Wikipedia to evaluate how F_{BOLT} varies as a function of climber's mass and fall factor, with the other parameters remaining constant as indicated in each figure. In Figure 20a, the fall factor (0.0-2.0) was varied, and in Figure 20b, the mass of the climber (40-120), keeping the other parameters as default.

A default pattern was assumed for the other variables in the plotting. Charts are also an indicator of how EM1 can be used to explore the values that F_{BOLT} may assume as each of its parameters varies. For specific results, appropriate calculations should be performed. The default values (i.e. assumed otherwise) were: climber's mass

(80kg); acceleration of gravity (9.807m/s2); fall factor (1.77); dynamic belay attenuation (0.70); last clip friction factor (0.34); maximum force in the UIAA test (9kN) (Mammut 9.8mm Transformer rope, 2017).

<table>
<tr><th colspan="4">Table 3 – Derivation of maximum impact force formula (F_{MAX}) for EM1 model according to classic derivation, added by EM1 attenuation improvements. The derivation uses conservation energy principle to extract an expression for F_{MAX}.</th></tr>
<tr><td>0</td><td>
</td><td>mg= climber's weight, local gravity
y_{MAX} = spring elongation as seen from vertical ax
L = open rope meterage
h = fall height
K = rope modulus
f = fall factor = h/L

Energy conservation at v=0:

TOTAL POTENTIAL ENERGY = ENERGY STORAGED IN THE SPRING

mg (h + y_{MAX}) = ½ • k • y^2_{MAX}</td><td>$$mg\cdot(h+y_{MAX})=\frac{1}{2}\cdot\frac{K}{L}y_{MAX}^2$$
$$mgh+mgy_{MAX}=\frac{1}{2}\cdot\frac{K}{L}y_{MAX}^2$$
Rearranging the equation
$$\frac{1}{2}\cdot\frac{K}{L}y_{MAX}^2-mg\cdot y_{MAX}-mgh=0$$</td></tr>
<tr><td>1</td><td>Using Hooke's Law:
$$F_{MAX}=k\cdot y_{MAX}=\frac{K}{L}\cdot y_{MAX}\quad\therefore y_{MAX}=\frac{K}{L}\cdot F_{MAX}$$</td><td>2</td><td>Replacing y_{MAX} by F_{MAX} in the equation and multiplying by K/L to simplify:
$$\frac{1}{2}\cdot F_{MAX}^2-mg\cdot F_{MAX}-\frac{K}{L}mgh=0$$</td></tr>
<tr><td>3</td><td>Solving for F_{MAX}, and making h/L=f (fall factor):
$$F_{MAX}=mg+\sqrt{(mg)^2+2Kf\cdot mg}$$</td><td>4</td><td>Simplifying:
$$F_{MAX}=mg\bullet\left[1+\sqrt{1+\frac{2Kf}{mg}}\right]$$</td></tr>
<tr><td>5</td><td>Adding EM1 proposal of attenuation for F_{MAX}, [Eq.4-6]:
$$F_{MAX}^{ME1}=mg\bullet\left[1+\sqrt{1+\frac{2Kf}{mg}}\right]\bullet R_{SD}^{MAX}$$</td><td>6</td><td>Finally, adding EM1 proposal for F_{BOLT}, [Eq.4-7]:
$$F_{BOLT}^{ME1}=mg\bullet\left[1+\sqrt{1+\frac{2Kf}{mg}}\right]\bullet(2-M)\bullet R_{SD}^{BOLT}$$</td></tr>
</table>

As only data from one experiment was found on DB, we should expect further testing to consolidate values. The challenge of assuring standardisation for the concept of dynamic belay must also be addressed. As an initial approximation, a rounding down of numbers from Czermin *et al.* is suggested as precaution, until some consensus emerges. In this study, as a preliminary analyses, the nominal values of 20% reduction in F_{MAX}, 30% in F_{BOLT} and 40% in F_{BEL} due to DB were assumed, compared with, respectively, 25.6%, 32.7%, and 42.0% from Czermin *et al.* On the other hand, a paper by Leuthäusser (2017) points to a remarkable 40% potential attenuation of dynamic belay. It is worth to remember that additional attenuations are common in a real climbing. A classical derivation of the EM1 equation is in Table 3, and Table 4 shows a synthesis of all equations studied and derived herein.

For the LCF, a widely used value is 0.34 (Petzl, 2005, apud Smith, 2017a, b) for the friction factor, which results in a reduction of F_{BOLT} of about 17% (the known factor of 1.66 derives from this assessment). Using Equation EM1 with these values, and the other values in the worst-case F_{MAX} formula, a "factor 2" fall (in fact, Q=f=1.77), we can now referentially compare the force on the last anchor with and without attenuation factors as proposed. If F_{MAX} for a rope is 9.0kN, a typical upper limit on current climbing ropes, and considering only LCF, F_{BOLT} would be F_{MAX}•1.66=15kN. Adding the DB attenuation, according to the EM1 model, we have:

F_{BOLT} = 9 • 1.66 • 0.7 = 10.5kN

A value well below 15kN, and far from the P-bolt resistance values, as calculated by Jimenez and Freitas (1999), considered undersized, of 1,300kgf or 12.8kN. This significant reduction in F_{BOLT} cannot be overlooked or ignored, otherwise, the security system is being stressed beyond reasonable costs. It also causes imbalance and inefficiency by paying high and unjustified prices for services and equipment due to the remarkable lack of real security needs. An analogy can be made without fear of overdoing it: putting on five seat belts will not increase anyone's safety, will artificially make the activity more expensive, and likely it will increase the risk rather than reduce it.

Table 4 – Equations of F_{MAX} and F_{BOLT} in the classic and in the semi-empirical, Experimental Model (EM1) version. A subscript zero means that the physical quantity comes from UIAA test of considered rope. DB=dynamic belay; Rxxx=reduction factor for "xxx" location due to dynamic belay effect; LCF=friction on to the last carabiner.

Reference name	Equation	Parameters	Referential values
Classic equation, Ref. Wikipedia (*Fall factor*)	**Maximum force on climber – Classic formula** $F_{MAX} = mg + \sqrt{(mg)^2 + 2mgKf}$	**f** – fall factor = **h/L** **F_{MAX}** – Maximum impact force on climber **m** – climber's mass **g** – gravity	A subscript zero means the value comes from UIAA test.
Classic equation in Wikipedia ("Wiki") format (*Fall factor*), where unknown K was replaced by published values (revised in 2019).	**Maximum force on climber – Wiki format (corrected) [Eq.2-10]** $F_{MAX} = mg + \sqrt{(mg)^2 + F_0(F_0 - 2m_0 g)\cdot\frac{g}{g_0}\cdot\frac{m}{m_0}\cdot\frac{f}{f_0}}$		**F_0**~7.5-10.5kN **g_0** – gravity in UIAA test = 9.807m/s² **f_0** =1.77 **m_0** = 80kg **g_0** = 9.807m/s²
EM1 equation derived from classic eq. for **F_{MAX}**, including LCF and (DB).	**Maximum force on climber – EM1 – Wiki format, [Eq.4-8]** $F_{MAX}^{EM1} = \left[mg + \sqrt{(mg)^2 + F_0(F_0 - 2m_0 g)\cdot\frac{g}{g_0}\cdot\frac{m}{m_0}\cdot\frac{f}{f_0}} \right] \bullet R_{MAX}^{DB}$		A subscript zero signals a value from UIAA test.
Equation Rope for rope modulus K, in case it is unknown, but data from UIAA test is not.	**Rope module – [Eq.2-9]** $K = \frac{F_0(F_0 - 2m_0 g_0)}{2m_o g_0 . f_0}$	Equations 14 (a&b) are in different formats. In Wiki's format, the rope modulus K, usually not known, is replaced. If you have K, it makes the calculation faster. Conversion from one format to other can be made with the formula at left.	A subscript zero signals a value from UIAA test.
EM1 equation derived from classic eq. for **F_{MAX}**, including friction factor of last carabiner (LCF) and DB.	**Maximum force on climber – EM1 – Original format, [Eq.4-9]** $F_{MAX}^{EM1} = mg \bullet \left[1 + \sqrt{1 + \frac{2Kf}{mg}} \right] \bullet R_{MAX}^{DB}$		Czermin *et al.* (2007) reduction factors: F_{MAX} –25.6% and F_{BOLT} –32.7%. This study suggests **$R(DB)_{MAX}$=0.80**
EM1 equation derived from the classic one for **F_{MAX}**, including DB. M is not a friction coefficient, but a net effect between rope and last carabiner (See 4.2.1)	**Maximum force on belayer – EM1 – Original format – [Eq.4-9]** $F_{BEL}^{EM1} = mg \bullet \left[1 + \sqrt{1 + \frac{2Kf}{mg}} \right] \bullet (1 - M) \bullet R_{BEL}^{DB}$		Ref. Czermin *et al.* (2007) F_{BEL} = –42.0% This study suggests **$R(DB)_{BEL}$ = 0.60** **M**=0.34, friction factor rope-carabiner (pub. on Petzl site (Smith, 2017a).
EM1 equation for **F_{BOLT}**, with LCF and DB	**Maximum force on bolt – EM1 – [Eq.4-7]** $F_{BOLT}^{EM1} = mg \bullet \left[1 + \sqrt{1 + \frac{2Kf}{mg}} \right] \bullet (2 - M) \bullet R_{BOLT}^{DB}$		This study suggests **$R(DB)_{BOLT}$=0.70**

5. CONCLUSION

- **Summary**

This paper sought to make a historical journey through the Physics of climbing in Portuguese language through the different flavours of the formula of maximum impact force in a fall in rock climbing, and herein translated to English. It used the idea of establishing a more complete formulation, called EM1 – Experimental Model 1 – for the maximum forces on the falling climber and on the last clip, from the first published classic model by Wexler (1950).

EM1 seeks to cover factors that can significantly affect the magnitude of F_{MAX} and F_{BOLT}, which are discussed and described, but do not appear in the classical formulas present in the scientific literature.

Furthermore, it aimed to achieve two secondary objectives: [a] to confront the proposed models of calculation, in fact variations of a basic, classical formula with the real phenomenon, seeking explanatory clues of the apparent contradictions between the values calculated by the classical formula, the resistance of the anchorages and the absence of accidents with failure of these anchorages in our country; and [b] to identify the important factors that are absent from classical theory, analyze the possibilities of quantification of these factors, and propose theoretical or empirical formulations that can consider the relevant factors in a practical manner.

The objectives were covered. The historical rescue of climbing Physics shows that the classical equation is older than most current climbers in the country suppose, as the authors Jimenez and Freitas (1999), in the only study on the forces in play conducted in the country and made available to the public. In it, the authors seem to ignore deductions published in 1950, when they redo them without methodological rigor. At the same time, the review showed that up to now, three factors of high influence in F_{MAX} and F_{BOLT}, the declivity of the route, the friction in the last clip and dynamic belay continue without standardization, definition and mathematical expression for assessment.

- **Criticism**

By means of the study of the various formulations, it is clear that, despite an effervescent discussion in various parts of the world about dynamic belay among many climbers of all styles, the UIAA to date does not explain or guide how exactly one should provide such a belay for a rope leader. Although there is a set of basic principles of general knowledge, a systematization of the process has not surfaced. It is worth saying that such a systematization is not necessary to climb, the sport is too diverse in environments and people to fit rigidly in any scheme, and there are not necessarily right or wrong solutions.

However, the UIAA has an intrinsic responsibility as the only international certifier. A standardization of dynamic belay, with explicitness of the basic attributes of a solid method and with good attenuation of forces, would impel the evolution and dissemination of quality belaying, and reduce the probability of accidents, a central goal for a given sport dancing and daring far up from the ground.

- **Answers**

The Experimental Model 1 proposes to explain phenomena that have the force to alter the magnitude of the forces in play, and to enrich the classical formulation with simple ways, but based on objective evidences, to assess these factors, which are the declivity the route or the sequence, the friction in the last clip and the dynamic belay. The detailed analysis of the classical formula with criticisms of its representativeness of the real phenomenon has an indirect benefit, which is to raise not usually focused questions for the readers, which directly affect the risks of this highly technical sport.

If the resistance of the P-bolts used by the overwhelming majority of rock climbers is, according to Jimenez and Freitas (1999), about 1,300kgf, how can the maximum force, according to the same authors, be "*close to 2,000kgf*"? Even considering that that study has methodological flaws, the question remains: if our P-bolts of more than half a century of services have half the resistance of the E-bolts (expansion) used in other countries, why is our record of accidents with bolt faults empty? The proposed formulation of EM1, in producing F_{BOLT} values below the resistance values of Jimenez and Freitas, opens an interesting explanatory possibility to answer the question that gave rise to this study. There are no accidents because the attenuations existing in a real fall reduce F_{BOLT} to much lower values than the classical theory infers.

- **Questions**

Although having included in EM1 the consideration of relevant and present factors in a real climb, I am aware that there are still issues to be explained and questions to be answered. For instance, this paper ignored two equally important, present phenomena that significantly affects the forces at play: the dry friction and the rope slack. The friction is considered in some papers as being responsible for as much attenuation as the LCF or DB. But of even more difficult quantification. The dry friction can also cause the fall factor to rise dangerously as the leader increases the number of clips and turns the rope does. It is not impossible that, as the climber thinks his falling factor is decreasing, it may be increasing. The focus of this paper was the formulas of F_{MAX} and F_{BOLT}, and dry friction is related to climbing safety in general, and will be considered later (Lima-e-Silva, 2019f, in press).

By knowing more about the forces at play and the factors that affect them, climbers can make better choices by consciously putting themselves at the level of risk they consider acceptable to themselves. If this paper can be of any help to a single climber to better choose his/her risks and climb with more awareness, and therefore to have more pleasure, it will have been worth it.

Like any study, this also clarifies some issues but raises numerous others. What is the best, practical, flexible and standard way of belaying? What are the values for the friction factors of modern, lighter and thinner carabiners? Why are there no records of critical anchor failure accidents with factor 2 falls? Why are factor 2 fall records so uncommon? Are there psychological factors, besides the physical ones, preventing falls before the first clip, or maybe it is just a probability issue?

On the one hand, we hope that field experiments conducted under the aegis of the scientific method will be carried out to answer these and other questions, and, on the other, that the vast experiences of Brazilian rock climbers can be collected, compiled and statistically analyzed, to disseminate this knowledge, something that seems to have not yet been done.

Left to right:
Sergio B. Pinto, Natanael de Oliveira, Alexandre Garcia, Pedro P. Lima-e-Silva (the author).
Summit, Brasilia chimney, Agulha Rock, Pancas, ES, Brazil. 1979.
A known beautiful and unsafe route.

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